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# THE EFFECT OF STRUCTURE-BORNE NOISE IN SUBMARINE HULL PLATING ON BOUNDARY LAYER STABILITY

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10 to the 4th power

field is decomposed into propagating wave fields and near-fields such as would surround foundation structures or other hull impedance discontinuities. It is found that for the frequency range generally associated with structure-borne noise, say 30 - $10^4$ Hz, the propagating wave fields are not destabilizing in that they do not contain components which are coincident with the appropriate eigenvalues of the Orr-Sommerfeld equation. By contrast, the near-fields will contain components which are coincident with the potentially destabilizing, eigenvalues of the Orr-Sommerfeld equation. These near-fields are interpreted in terms of equivalent propagating fields to which they are found to be of (relatively) low level. Available experimental results are discussed in terms of the submarine environment and it is found that although transition may be affected by structure-borne noise levels less than, say, the expected free-stream turbulence levels, the measured noise levels required for destabilization are higher than one expects in submarine hull plating.

Finally, a mechanical analogue to the Orr-Sommerfeld equation is presented in the form of a vibrating elastic plate resting on a locally reacting foundation of specific form. In the analogue Reynolds number becomes the cube of the slenderness ratio of the plate. Some aspects of the solutions to the Orr-Sommerfeld equation, as it is applied to boundary layer transition, are interpreted in terms of the analogue system.

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## SUMMARY

This report deals with the potential boundary layer destabilizing action of machinery generated structure-borne noise in submarine hull plating. Boundary layer transition is assumed to be governed by the eigenvalues of the Orr-Sommerfeld equation and the structure-borne noise field is taken to be flexural in nature. The structure-borne noise field is decomposed into propagating wave fields and near-fields such as would surround foundation structures or other hull impedance discontinuities. It is found that for the frequency range generally associated with structure-borne noise, say 30 - $10^4$ Hz, the propagating wave fields are not destabilizing in that they do not contain components which are coincident with the appropriate eigenvalues of the Orr-Sommerfeld equation. By contrast, the near-fields will contain such components. These near-field effects are interpreted in terms of equivalent propagating fields to which they are found to be of (relatively) low level. Available experimental results are discussed in terms of the submarine environment and it is found that although transition may be affected by structure-borne noise levels less than, say, the expected free-stream turbulence levels, the measured noise levels are higher than one expects in submarine hull plating.

Finally, a mechanical analogue to the Orr-Sommerfeld equation is presented in the form of a vibrating elastic plate resting on a locally reacting foundation of specific form. In the analogue Reynolds number becomes the cube of the slenderness ratio of the plate. Some aspects of the solutions to the Orr-Sommerfeld equation, as it is applied to boundary layer transition, are interpreted in terms of the analogue system.

## I. INTRODUCTION

Early studies of boundary layer control were concerned primarily with airfoils where suction was found to be a promising control measure.[1] Heating of the boundary was first studied for gases, for which a temperature rise corresponds to an increase in viscosity, and consequently to an increase in boundary layer thickness. Heating was therefore considered as a destabilizing factor.[2] The opposite effect is achieved in water, where an increase in temperature reduces the kinematic viscosity and consequently, the boundary layer thickness. The shift of emphasis to boundary layer control in water coincided with the discovery by Kramer, in 1960, that a compliant boundary, specifically a rubber membrane, can delay transition from laminar to turbulent flow.[3]

The application of boundary layer control techniques to submarines in particular is desirable since boundary layer control would not only reduce drag, but also turbulence-generated self-noise which masks sonar signals at the higher speeds. Furthermore, the reduced drag entails reduced propulsive power requirements. This, in turn, permits a smaller hull displacement, reduced auxiliary power requirements, and thus a reduction of machinery-generated radiated noise as well. Problems that may be encountered in the successful application of boundary layer control techniques to submarines concern the nature of the environment in which the submarine operates. That is, the possible destabilizing effects of free-stream turbulence, acoustic disturbances, hull vibrations, etc.

Structure-borne noise has no effect on drag in current submarines, because most of the hull plating, and in fact, all of the pressure hull plating not concealed by thin plating, is exposed to the turbulent boundary layer at all speeds. However, if the boundary layer is controlled by, say, heating and suction, the vibrating pressure hull plating will be in contact with the laminar boundary layer. This report, in particular, details an investigation into the potential boundary layer destabilizing action of machinery generated structure-borne noise in submarine hull plating, especially the effect of flexural waves in submarine hull plating on transition as modelled by the Orr-Sommerfeld equation.

## II. THE CHARACTERISTICS OF STRUCTURE-BORNE NOISE IN SUBMARINE HULLS

Machinery generated structure-borne noise in operational submarines consists of tonals caused by, for example, blade passage in pumps as well as broad-band components caused by, for example, turbulent flow in piping systems. In all cases the excitation levels are such that the vibration response of the hull plating is the result of a linear phenomenon. The spectrum of machinery generated structure-borne noise can be considered to range from 30 Hz to 10 kHz.

Machinery induced vibratory forces are transmitted through a foundation structure to the hull, which responds to these excitations primarily in flexure. Freely propagating flexural waves in a plate exhibit dispersion. They display a phase velocity, in vacuo

$$c_f = \frac{(c_p h\omega)^{\frac{1}{2}}}{12^{\frac{1}{4}}} \qquad (1)$$

where $c_p$ = compressional wave velocity in plates
$h$ = plate thickness
$\omega$ = frequency in radian/sec.

For completeness, it should be mentioned that at the lower frequencies this result is modified somewhat by the effect of water loading which tends to reduce the velocity. Also, at the higher frequencies, the above equation must be modified to account for rotary inertia and shear effects. However, both corrections are not significant to the present task.

Hull plating typically ranges in thickness from 5 cm. (2 in.) for the pressure hull to 1 cm. (3/8 in.) for the faired portions of the hull. The overall charcteristics of the vibration field that is associated with the hull plating may be conveniently summarized by plotting the appropriate phase-velocity vs. frequency relationship for relevant parameters. This is shown in Fig. 1. It can be verified that the flexural wave velocity in the pressure hull plating exceeds the free stream flow velocity over the entire range of interest. Furthermore, it is noted that the flexural wave velocity exceeds the sound velocity in water only above the coincidence frequency, which is approximately 5 kHz for representative 5 cm. (2 in.) pressure hull plating.

In addition to freely propagating waves in the hull plating, localized, or near-field, vibrations are present in the immediate vicinity of an equipment-hull interface as well as at any major discontinuity in the impedance of the hull structure. At low frequencies major hull discontinuities are limited to bulkheads, but at higher frequencies small frames also provide significant discontinuities to the hull plating. The most significant difference between these near-field vibrations and propagating waves is that for a given frequency the near-field vibrations contain a continuous phase velocity, or wavenumber, spectrum. If one models this effect as a uniform line excited plate of infinite extent, or equivalently a point excited beam, then the phase velocity spectrum of the plating response at the drive point is as shown in Fig. 2 for a representative value of structural damping given by $\eta = 0.1$.[4] Thus, although the near-field contains phase velocity components $c_\gamma$, such that $c_f/c_\gamma >> 1$, the magnitudes of these components are low relative to the component that corresponds to the characteristics of freely propagating waves in the plating, i.e., $c_f/c_\gamma = 1$.

## III. BOUNDARY LAYER STABILITY; TRANSITION

The classic linear analytical approach to the stability of two dimensional parallel flows in general, and boundary layers in particular, is to solve for the eigenvalues of the Orr-Sommerfeld equation.[5] The Orr-Sommerfeld equation results from the assumption of a disturbance of the form

$$\Phi = \phi(y) \exp[i\alpha(x - ct)]$$

where x is the coordinate along the direction of flow. The nature of the eigenvalues, viz., $(\alpha, c)$, is then used to determine the condition of stability of the flow, i.e., whether the flow can support a disturbance of a given form without producing an exponentially amplified response. For example, spatial amplification is associated with negative $\alpha_i$. The results of these analyses are typically presented in the form of stability contours plotted on graphs whose abscissa is the Reynolds number and whose ordinate is the, non-dimensionalized, disturbance wavenumber for various phase velocities of the disturbance (Fig. 3). Alternatively, contours may be plotted on graphs of frequency $\omega$ ($=\alpha c$) vs. Reynolds number (Fig. 4). The neutral stability contour is defined as the contour that corresponds to $\alpha_i = 0$. However, observations have shown that in many instances the criterion of neutral stability, i.e., that transition occurs at the minimum Reynolds number capable of sustaining a neutrally stable disturbance, is too conservative. Thus, the "$e^n$" rule has been developed.

The "$e^n$" rule is an empirically based criterion for applying linear stability theory to the problem of predicting boundary layer transition.[6,7] For nearly parallel flow, local amplification factors, i.e., values of n, for a given disturbance frequency are obtained from the solutions of the Orr-Sommerfeld equation and integrated in the flow direction from the point of neutral stability.[8] If the disturbance whose frequency yields the maximum amplification factor at transition is assumed to, in fact, cause transition, then it is found that the value of the amplification factor itself, i.e., the quantity

$$[n \equiv \int_{x_o}^{x_t} \alpha_i(x)\,dx],$$

is reasonably constant and corresponds to $n \simeq 9$.

Regardless of the value of n, the point is that transition is associated with disturbances whose wavenumber and phase-velocity are such that they coincide with the neutral stability curves, i.e., $n = 0$, or else they penetrate this curve to some degree, i.e., $n > 0$. This fact will constitute the basis for assessing the impact of hull vibrations on transition.

## IV. STRUCTURE-BORNE NOISE AS A POTENTIAL DESTABILIZING AGENT

Destabilization of a laminar boundary layer by structure-borne noise requires (1) a transfer mechanism for the disturbance to interact with, or "penetrate," the boundary layer and (2) once penetrated, an instability mechanism to initiate amplification of the boundary layer's response to the disturbance.

Four different configurations through which structure-borne noise in submarine hulls represent a potential destabilizing agent are shown in Fig. 5. Structure-borne noise in uncoated pressure hull plating (Item 1 in Fig. 5) represents a potential disturbance which is introduced directly into the boundary layer. The drive point impedance of the relatively thick pressure hull plate is sufficiently high so that the plating as seen by the boundary layer is effectively rigid. In other words, the pressure-hull plating can be assumed to exhibit an infinite internal impedance. Thus the relevant, linearized, analysis requires the solution to the inhomogeneous Orr-Sommerfeld equation with the inhomogeneity being a prescribed velocity distribution at the boundary. The importance of this circumstance is that, although the actual response of the boundary layer, e.g., amplification factors, etc., will be a function of the inhomogeneity, the instability mechanisms, i.e., the eigenvalues of the Orr-Sommerfeld equation, correspond to the homogeneous, i.e., rigid boundary, problem. Thus, the criterion for destabilization is coincidence of the disturbance wavenumber-phase velocity of wavenumber-frequency characteristics with the eigenvalues corresponding to the classical Tollmien-Schlichting instabilities (see Appendix A).

### A. The Effect of Propagating Waves on Transition

In view of the above stated requirement for destabilization the question as to whether propagating waves in submarine plating represent a potential destabilizing factor reduces to the question of whether these waves are coincident with the eigenvalues of the Orr-Sommerfeld equation. Fortunately, for the parameters of interest coincidence does not occur. This can be seen by comparing Figs. 1 and 3. From Fig. 3 it can be seen that Tollmien-Schlichting instabilities, for a Blasius profile, exhibit phase velocities which are bounded by $c_{TS}/U_\infty \le 0.5$.[9] In order to compare the results of Figs. 1 and 3, it is convenient to express this velocity as follows:

$$c_{TS} \le 0.5\, U_\infty = 0.5 c_a M = 0.5\, c_f (c_a/c_f) M = [0.5\, M(\omega_c/\omega)^{\frac{1}{2}}] c_f \qquad (2)$$

where M is the Mach number and $\omega_c$ $(= 12^{\frac{1}{2}} c^2/(hc_p))$ is the critical frequency of the hull plating in water. Thus, for coincidence between the phase velocity of the flexural waves in the plating with that of T-S waves

$$\omega \le (0.5M)^2 \omega_c$$

For a speed of 80 knots, this requirement for the pressure hull plating implies a frequency less than 1 Hz, which is outside our frequency range of interest. For completeness it is noted that this conclusion is not changed if, instead of the Blasius profile, one uses the profiles expected for heated boundary layers as calculated by Rockwell.[10]

Item 3 in Fig. 5 poses a different circumstance in that here the structure-borne noise field does not interface with the boundary layer, but rather is isolated by means of a compliant, anti-radiation, coating. This differs from the previous situation since here the boundary/boundary-layer interface, by design, exhibits a relatively low impedance. This means that a feedback loop is indeed possible between the boundary layer and the boundary. In other words the non-rigid boundary can modify the eigenvalues of the Orr-Sommerfeld equation and in fact introduces additional sets of eigenvalues.[9] It must therefore be determined if these modified eigenvalues, i.e., instabilities, can be excited by the structure-borne noise field associated with the pressure hull plating. An analysis of this situation is presented in Appendix B where it is again shown that for the relevant parameters the conditions for destabilization are not satisfied by present day submarine constructions.

Item 2 in Fig. 5 falls somewhere between Items 1 and 3. However, in lieu of the previous results it is concluded that structure-borne noise in the faired hull plating, in the form of propagating waves, as with the pressure hull plating, is not a potential destabilizing agent. Finally, for completeness, disturbances associated with Item 4 in Fig. 5, propagating acoustic waves of moderate level also fall outside the wavenumber/phase-velocity range of potential instabilities as calculated via the Orr-Sommerfeld equation.

It should be noted that in addition to the mechanisms of modifying boundary layer stability indicated in Fig. 5, a mechanism based on vortex generation by an acoustic field is suggested by the work of Mechel and Schilz.[11] This mechanism requires a boundary layer flow along a surface that has discontinuities in the form of sharp edges. The oscillating flow associated with an intense sound field can establish an alternating vortex structure when the acoustic motion is normal to a sharp edge. The vortex motion then couples in a nonlinear manner with the mean flow to cause higher frequency components of the disturbance to be generated. The possibility therefore can not be dismissed that the intense acoustic motion results in flow disturbances that can match either constructively or destructively the incipient Tollmien-Schlichting disturbances. It should be noted that this sharp edge mechanism would not be expected to be present on a smoothly faired shape such as that of a submarine.

## B. The Consequences of Near-Field Effects on Transition

As discussed earlier, near-field vibrations differ from the propagating field in that the near-field contains a continuous wavenumber, or phase velocity, spectrum. This means that the near-field will, in fact, contain a component that is coincident with T-S waves. Therefore, here the question as to whether near-field effects in submarine plating represent a potential destabilizing factor reduces to a question of level. For hull plating in the immediate vicinity of an equipment foundation or for substantial impedance discontinuities, the overall near-field, i.e., non-propagating levels, can be expected to be of the same level as the propagating wave levels. The relative magnitudes of those phase velocity components of the near-field that may be coincident with T-S waves can be obtained from Fig. 2. Note that the excitation spectrum level decreases monotonically with a decrease in phase velocity from the point corresponding to the phase velocity of propagating waves.

A measure of the potential destabilizing effect of this near-field may be obtained by relating it to an "equivalent" propagating field. The meaning of equivalent is equal wavenumber and phase velocity. As an example consider the flexural response of thin hull plating at the high end of the frequency range of interest, say 10 kHz. From Eq. 2, at this frequency and assuming a speed of 80 knots, coincidence occurs at $c_\gamma/c_f \equiv c_{TS}/c_f \simeq 2 \times 10^{-2}$. From this value the spectrum level continues to decay at a rate proportional to $(c_\gamma/c_f)^4$. Referring to Fig. 2 the level of this component is down by a factor of $1.6 \times 10^{-8}$ from the level of the propagating component. Thus, the near-fields surrounding local excitations or discontinuities represent a comparable threat to stabilization as the equivalent propagating field whose level is down by a factor of $1.6 \times 10^{-8}$ or 156 dB.

## V. EXPERIMENTAL DATA RELEVANT TO THE PROBLEM OF STRUCTURE-BORNE NOISE EFFECTS ON TRANSITION

### A. Destabilization with Coincidence

In this section the salient features of an experimental study of the effects of boundary vibrations on transition are presented. The work was performed by Schilz in 1969.[12] Even though this work was funded by the U.S.A.F. European Office of Aerospace Research and published, albeit in German, it appears to have attracted little notice in the U.S.

#### 1. Description of Tests

It will be found in these tests that, as discussed in Section IV., the effects of flexural waves propagating in a plane boundary are most significant if the disturbance is coincident with a T-S instability. The T-S phase velocity lies between 1 and 20 m/s in Schilz's tests. No practical structural material can achieve these low velocities in the frequency range of interest. Schilz overcame this difficulty by using a transducer array wherein adjoining elements are driven out-of-phase. One pair of elements thus simulates one wavelength. Since the spacing of the transducers in a given test is invariant, and can only be varied discontinuously between tests, increasing values of $U_\infty$ are associated in each test with rising frequencies. The boundary condition thus achieved displays, not surprisingly, a strong harmonic content.

The displacement amplitudes of the vibrating boundary were measured with a capacitance probe. Hot wire anemometers were used to measure the flow velocities. The tests were performed on a flat plate with a sharp leading edge. Unless tripped, the laminar boundary layer extends over the entire plate (0.5 m).

#### 2. Experimental Results

Destabilization occurs primarily within the neutral stability curve of T-S waves, but was occasionally observed at lower frequencies. Only semiquantitative measurements could be obtained for the threshold amplitude of flexural waves required to trip the boundary layer for different values of $U_\infty$. It was found that for the coincidence conditions, the amplitudes required to affect the boundary layer are much smaller ($10^{-2}$ millimeters or less) than the boundary layer displacement thickness (0.4 millimeters). The results show a marked decrease with increasing $U_\infty$.

A marked rise of the threshold occurs on both sides of the coincidence frequency, defined as the frequency for which the effective phase velocity of the "flexural wave" matches the T-S velocity. When the disturbance is in the form of a band of noise, only that band which contains the coincidence frequency shifts the transition point upstream.

3. Conclusions

The principal outcome of the experiments is that the conclusion drawn in Section IV., namely that it is improbable that machinery generated structure-borne noise in the form of propagating waves could destabilize a laminar boundary layer, appears to be verified.

Another conclusion, drawn by Schilz and not unrelated to the problem at hand, is that structure-borne noise may provide a means for actually suppressing turbulence, i.e., delaying transition. For this hypothesis, Schilz relies heavily on Benjamin's analysis.[13] Benjamin defines a type A instability as the T-S wave modified by the presence of a flexible boundary. He shows that this is an energy-deficient wave mode. Damping at a flexible boundary further enhances this energy deficiency, thus leading to energy flow from the main stream to the boundary layer, and consequently further destabilization. Schilz postulates that instead of supplying the needed energy by diverting it from the main stream, as suggested by Benjamin, this energy can be provided by actively exciting flexural waves in the boundary. These waves convey energy to the boundary layer, thus compensating for energy dissipation in the T-S waves. The flexural waves thus play the role of negative damping, which can be anticipated to have a stabilizing effect, since positive damping exerts a destabilizing influence. At coincidence, the condition for energy flow from the wall into the fluid depends on the phase. (Becker[14] concludes that for any stable inviscid velocity profile, energy flow is from the fluid to the vibrating boundary. This can presumably be assimilated to positive damping by the boundary, which Benjamin showed to be destabilizing.

It would appear the Schilz's technique of forcing energy to flow from the wall into the boundary layer can only be implemented *for unstable profiles.)*

Schilz finds that he does indeed achieve stabilization over a broad phase angle $\Delta\phi$ centered approximately on $180^o$, i.e., for phase opposition between T-S waves and the wall vibrations. These results were obtained as follows: the boundary layer is tripped to generate T-S waves, the resulting transition point being located downstream of the flexural array. The array is then energized to produce flexural waves whose velocity coincides with that of the T-S waves. Appropriate amplitudes and $\Delta\phi \simeq 180^o$ can lead to stabilization. For a $180^o$ phase angle, an increase in the flexural wave amplitude first increases the critical Reynolds number and, for amplitudes exceeding an optimal amplitude $3 \times 10^{-6}$ meters for $U_\infty$ = 27.4 m/s, reduces it. If the boundary layer is not tripped, these extremely low flexural wave amplitudes have a destabilizing effect, presumably because they are in phase with the T-S waves they generate, rather than being out of phase with the T-S waves generated by the tripping wire.

## B. Destabilization Without Coincidence

In a separate paper Schilz discusses non-linear effects that may be important when coincidence is not achieved.[15] A more detailed description of this type of circumstance, i.e., non-coincidence, can be found in the reports by Norair,[16] documenting their preliminary wind tunnel experiments on the effects of noise on transition.

### 1. Description of Tests

A 4-percent-thick straight laminar suction wing of 17-foot chord was investigated in the Norair 7- by 10-foot low turbulence wind tunnel at $\alpha = 0^o$ angle of attack in the presence of external sound, and in addition, panel vibration. The sound consisted of discrete frequencies and octave bands of random noise in the 150 to 4000 cps frequency range, while the vibration frequencies were 100, 190, and 1290 cps.

Considerations of the type of structure contemplated by Norair indicated that peak panel accelerations of about 10g in the 100 to 1000 cycle frequency range could be expected. Thus, to answer the question as to the potential influence of mechanical vibrations of an external wing surface on the behavior of a laminar (suction) wing, a high-speed, d.c. motor-driven, reciprocating mass shaker was attached to the wing. The motor power-limited the shaker to a frequency of about 260 Hz and a 46 g peak panel acceleration for intermittent operation.

2. Experimental Results

In all instances of 10g peak acceleration or more, suction had to be increased above the minimum drag value to maintain full chord laminar flow. For a peak panel velocity ratio $v/U_\infty = 2.0 \times 10^{-3}$, the maintenance of full chord laminar flow by raising suction level required a 24 percent increase in suction coefficient and a corresponding 5 percent increase in equivalent drag, compared to a case with no vibration. Since panel velocity ratios, $v/U_\infty$, in the order of $2 \times 10^{-3}$ are possible for thin straight wing laminar suction aircraft, it was concluded that some additional suction might be required.

To interpret the magnitude of the vibration levels in the Norair tests to the problem of structure-borne noise in submarine hull plating, consider the example of a machine with, say, a motor imbalance that produces a 1 lb. vibratory force on the hull plating. If one crudely models the hull plating as an infinite elastic plate in order to obtain an estimate of its impedance, then 2 inch plating yields a drive point impedance $z \sim 10^3$ lb./in./sec. Thus, the excitation will produce hull accelerations of approximately

$$a/g = \omega v/g = 2\pi(1)(10^{-3}/386)f$$

$$\approx 1.5 \times 10^{-5} f$$

where f is the excitation frequency in Hz. If one considers 5 kHz the highest frequency of interest the maximum plating acceleration levels are given by $a/g \approx 10^{-1}$ compared to $a/g \approx 10$ in the Norair tests.

This particular result is somewhat disturbing in that although destabilization required structure-borne noise levels two orders of magnitude greater than one expects in submarine hull plating, the mechanism behind the destabilization process appears to be relevant.

For example, the structure-borne noise levels in the Norair test, as well as in an operational submarine, are small in terms of both the boundary layer thickness and the characteristic structural dimensions, viz. plating thickness and length. Unforturnately, at present the available Norair documentation covers their preliminary results. If, in fact, their work or similar work is continuing, it would be most relevant to the problem at hand.

## VI. A MECHANICAL ANALOGUE TO BOUNDARY LAYER STABILITY

### A. The Analogue System

During the course of this study a mechanical analogue to the Orr-Sommerfeld equation was discovered. This analogue was pursued and the results are presented in this section. A qualitative description of some features of the submarine structure-borne noise problem is presented, i.e., interpreted, in terms of the analogue system in Section B.

Consider the steady state vibrations of an elastic plate. For present purposes, the plate is taken to be of infinite extent along the x direction, of "characteristic" width $\ell$ and thickness h (Fig. 6). The partial differential equation governing the vibration response of a plate can be obtained from the Timoshenko-Mindlin model of plate vibrations.[17] This model allows, for the same structural wavelength, two different modes of wave propagation corresponding to two different ratios of shear to bending motion. Or, stated differently, for the same frequency this model allows two different modes of wave propagation corresponding to two different structural wavelengths. The model is generally considered valid up to frequencies that produce structural wavelengths $\lambda_s$ such that $(\lambda_s/h) \gtrsim O(10)$.

Using the rotation of the plating cross-section in the y-z plane, $\gamma(x,y)$, as the response parameter of the plate, the governing partial differential equation of motion may be expressed as

$$[\nabla^2 + \Omega^2][\nabla^2 + (c_d/c_s)^2\Omega^2]\gamma(x,y) - \beta^2\{\Omega^2\gamma(x,y) + \beta[\nabla^2 + (c_d/c_s)^2\Omega^2]M(x,y)\} = 0 \tag{3}$$

where

$\nabla^2 = (\partial^2/\partial x^2 + \partial^2/\partial y^2)$

$x,y$ = coordinate axes nondimensionalized to $\ell$

$\ell$ = characteristic plate dimension along the y axis

$h$ = plate thickness

$\beta$ = slenderness ratio of plate = $\sqrt{12}(\ell/h)$

$c_d/c_s$ = ratio of dilatational to shear wave speed in plate

$\Omega$ = nondimensionalized frequency = $\omega\ell/c_d$ where $\omega$ is the circular frequency

$Q(x,y)$ = distributed lateral force acting on the plate, nondimensionalized to $12E/(1-\nu^2)$

$M(x,y)$ = distributed moment acting on the plate, nondimensionalized to $12E\ell/(1-\nu^2)$

$E,\nu$ = elastic modulus and Poisson's ratio of plating material

Now consider the case where the lateral forces and moments, Q and M, represent the response of a homogeneous and isotropic, but locally reacting, foundation on which the plate is resting (Fig. 6). This may be expressed in terms of the foundation's impedances $Z_Q(y)$ and $Z_M(y)$

$$
\begin{aligned}
Q(x,y) &= -i\Omega Z_Q(y)\gamma(x,y) \\
M(x,y) &= -i\Omega Z_M(y)\gamma(x,y)
\end{aligned}
\tag{4}
$$

Further, the assumption is made that for the plate under consideration

$$(c_d/c_s)^2 = 1 \tag{5}$$

Although this is not ordinarily the case for homogeneous plating materials, one can envision this being accomplished by means of a sandwich-like plate construction.

Substituting Eqs. 4 and 5 into Eq. 3 yields

$$
\begin{aligned}
&[\nabla^2 + \Omega^2][\nabla^2 + \Omega^2]\gamma(x,y) - \beta^2\{\Omega^2\gamma(x,y) - i\Omega\beta[\partial[Z_Q(y)\gamma(x,y)]/\partial y \\
&\quad + (\nabla^2 + \Omega^2)[Z_M(y)\gamma(x,y)]]\} = 0
\end{aligned}
\tag{6}
$$

It follows that the wave number spectrum in the x direction of the plate's response, as given by the Fourier transform of Eq. 6, satisfies the equation

$$\tilde{\gamma}''''(y) - 2\eta^2\tilde{\gamma}''(y) + \eta^4\tilde{\gamma}(y) = \beta^2\{\Omega^2\tilde{\gamma}(y) - i\Omega\beta[[Z_Q(y)\tilde{\gamma}'(y)] + [Z_M(y)\tilde{\gamma}(y)] - \eta^2[Z_M(y)\tilde{\gamma}(y)]'']\} \tag{7}$$

where

$$\tilde{\gamma}(y) = \int_{-\infty}^{\infty} \gamma(x,y)\exp(ik_x s)\,dx,$$

$$\eta^2 = (k_x^2 - \Omega^2)$$

and the "prime" notation is used to represent *differentiation with respect* to y. At this point the form of the foundation impedances are restricted to take the following form:

$$\text{(i)} \quad Z_Q(y) = -2Z_M'(y) \tag{8}$$

and

$$\text{(ii)} \quad \beta|Z_M''(y) + \eta^2 Z_M(y)| >> \Omega \tag{9}$$

Thus Eq. 7 becomes

$$\tilde{\gamma}''''(y) - 2\eta^2\tilde{\gamma}''(y) + \eta^4\tilde{\gamma}(y) = i\Omega\beta^3\{[\eta^2 Z_M(y) + Z_M''(y)]\tilde{\gamma}(y) - Z_M(y)\tilde{\gamma}''(y) \tag{10}$$

*or*

$$\tilde{\gamma}''''(y) - 2\eta^2\tilde{\gamma}''(y) + \eta^4\tilde{\gamma}(y) = -i\eta\beta^3\{[\eta^2 Z_M^{*''}(y)]\tilde{\gamma}(y) - Z_M^*(y)\tilde{\gamma}''(y) \tag{11}$$

where

$$\begin{aligned} Z_M^*(y) &= -Z_M(y)/[(k_x/\Omega)^2 - 1]^{\frac{1}{2}} \\ &= -(\Omega/\eta)Z_M(y) \end{aligned} \tag{12}$$

The solutions to Eq. 11 yield the eigenvalues ($\eta$) as a function of $\beta$ and $Z_M^*(y)$.

Eq. 11 will now be compared to the Orr-Sommerfeld equation which, for completeness, is written below

$$\phi''''(y) - 2\alpha^2\phi''(y) + \alpha^4\phi(y) = -i\alpha R\{[(U(y) - c)\alpha^2 + U''(y)]\phi(y) - (U(y) - c)\phi''(y)\} \tag{13}$$

$\phi(y)$ is defined by the perturbation stream function $\Phi(x,y)$ which is assumed to be of the form

$$\Phi(x,y) = \phi(y)\exp]i\alpha(x - ct) \tag{14}$$

x and y are cartesian coordinates parallel and normal to the direction of flow, U(y) describes the main flow, and R is the Reynolds number.

In dealing with boundary layers, U(y) represents the mean velocity profile within the boundary layer and all equations can be non-dimensionalized whereby lengths are normalized to the boundary layer thickness $\delta$; velocities to the free stream velocity $U_\infty$, time by $\delta/U_\infty$ and pressure by $\rho U_\infty^2$ where $\rho$ is the fluid density.

For a given main flow profile the eigenvalues of the Orr-Sommerfeld equation, as a function of Reynolds number, are given by the wavenumber and phase velocity set ($\alpha$,c) that satisfies Eq. 13. Referring to Eq. 14 it is noted that $\alpha_i < 0$ denotes spatially amplified disturbances with increasing values of x. The subscript "i" is used to denote the imaginary component.

A comparison between Eqs. 12 and 13 reveals the following formal analogy (Table 1).

Table 1 - Mechanical Analogue to Parallel Flow Stability

| Parallel Flow Stability (Orr-Sommerfeld Model) | Plate Vibrations (Timoshenko-Mindlin Model) |
|---|---|
| $\delta$ | $\ell$ |
| $x, y$ | $x, y$ |
| $\phi(y)$ | $\tilde{\gamma}(k_x, y)$ |
| $\alpha$ | $\pm\eta$ |
| $R$ | $\beta^3$ |
| $U(y)-c$ | $\pm Z_M^*(y)$ |

It is interesting to note the following:

1. Reynolds number (R) is analogous to the cube of the slenderness ratio of the plate ($\beta$) whereby large Reynolds number corresponds to slender plating.

2. The inviscid components of the Orr-Sommerfeld equation are modelled by the plating while the mean-flow related, or viscid, components are modelled by the locally reacting foundation upon which the plating rests.

3. The critical layer, that is, the value of y across which the real component of the quantity $[U(y) - c]$ changes sign, corresponds to the value of y in the analogue across which the real component of the foundation impedance $[Z_M^*(y)]$ changes sign.

4. Neutral stability, whereby $\alpha$ and c are purely real quantities, corresponds to a completely conservative analogue system with the stipulation that $k_x > \Omega$.

To complete the formal analogy the appropriate boundary conditions must be included. This appears to be a potential strength of the analogy since a vibrating plate can, in general, admit passive, as well as active, and conservative as well as dissipative, terminations. Thus, for example parallel flows between general impedance boundaries should be amenable to the analogue. However, for present purposes, it is merely noted that the classical boundary conditions for boundary layer flow over an effectively rigid plane, which are given by

$$\phi(0) = \phi'(0) = 0 \tag{15a}$$

and

$$\lim_{y\to\infty} \phi(y) = \lim_{y\to\infty} \phi'(y) = 0 \tag{15b}$$

are modelled in the analogue by a plate that is semi-infinite in extent along the y axis and cantilevered from an effectively rigid termination at the y = 0 boundary, i.e.,

$$\tilde{\gamma}(k_x,0) = \tilde{\gamma}'(k_x,0) = 0 \tag{16a}$$

and

$$\lim_{y\to\infty} \tilde{\gamma}(k_x,y) = \lim_{y\to\infty} \tilde{\gamma}(k_x,y) = 0 \tag{16b}$$

Thus in theory, although it is not necessarily proposed here, if one were to construct such a structure as is described above, and impulse excite it such that all frequencies and wavenumbers were excited, then the plate's response would contain the solutions to, i.e., the eigenvalues of, the Orr-Sommerfeld equation.

## B. Stability Phenomena in the Analogue System

In the previous section a formal analogue to the Orr-Sommerfeld equation was presented. In this section some features of the solutions to the Orr-Sommerfeld equation, when applied to boundary layers, are interpreted in terms of the analogue system. However, it is noted that any application of these interpretations to experimental observations of transition presupposes that the mechanisms governing the observations are, in fact, captured by the linearized Orr-Sommerfeld theory. It is also noted that in this section the spatial amplification formulation to the solutions of the Orr-Sommerfeld equation will be used. That is, the wavenumber will be considered to be complex, $\alpha = \alpha_r + i\alpha_i$, and frequency will be taken to be real $\omega = \alpha c = \omega_r$. This leads to a spatial amplification factor of the form $\exp[-\alpha_i x]$ and thus negative values of $\alpha_i$ yield amplification and positive values of $\alpha_i$ correspond to attenuation.

### 1. Reduced Analogue for Divergence

"Divergence" as opposed to "flutter" is the particular form of instability whereby wave amplitude varies monotonically with position. Divergence corresponds to solutions of the Orr-Sommerfeld equation with purely imaginary values of $\alpha$, i.e., $\alpha = i\alpha_i$. In the analogue system this implies that $\eta$ is purely imaginary and thus $(k_x/\Omega) < 1$. In fact it is of particular interest to consider that this condition be satisfied in the limit $(k_x/\Omega) = 0$. Limiting ourselves to this situation the corresponding reduction in the analogue is shown in Table 2.

To interpret this reduced analogue consider the analogue system in Table 1 as a two-dimensional structural waveguide, albeit leaky along the y axis. Since the plating itself is uniform the extent of this "leak" will depend on the variations in $Z_M^*(y)$, with strong variations tending to reflect rather than transmit energy and thus minimize the leakage. Each vibration mode of such a system, i.e., a mode with a given mode shape in the y direction, will generally exhibit a high-pass cut-off frequency, i.e., a frequency below which the mode cannot freely propagate along the

Table 2 - Reduced Analogue for Boundary Layer Divergence

| Boundary Layer Stability (Orr-Sommerfeld Model) | Beam Vibrations (Timoshenko-Mindlin Model) |
|---|---|
| $\delta$ | $\ell$ |
| $y$ | $y$ |
| $\phi(y)$ | $\tilde{\gamma}(0,y)$ |
| $\alpha$ | $\pm i\Omega$ |
| $R$ | $\beta^3$ |
| $U(y)-c$ | $\pm Z_M^*(y)=\pm iZ_M(y)$ |

x-direction. This frequency will be the natural frequency of the corresponding standing wave mode of the waveguide cross-section along the y axis,[18] and thus may be calculated by setting the x-direction wavenumber ($k_x$) equal to zero in the equation of motion (Eq. 11). Therefore, the wavenumber ($\alpha$) of the divergent instability associated with a given phase velocity (c) may be interpreted as the cut-off frequency ($\Omega$) of the analogue system associated with a given foundation impedance $Z_M(y)$. The reduced analogue shown in Table 2 contains a number of interesting features:

1. Physically, the dimensionality of the analogue has been reduced to that of a beam (in the y-z plane) rather than a plate.

2. Real values of c imply a purely imaginary foundation impedance $Z_M(y)$. This implies a non-dissipative foundation and thus the mechanical analogue is a purely conservative system.

3. Assuming frequency to be a real positive quantity, the positive sign for $Z^M(y)$ implies $\alpha = +i\Omega$, and thus in view of Eq. 14, corresponds to stability. Similarly, the negative option for $Z^M(y)$ corresponds to an analogy where $\alpha = -i\Omega$ and yields the unstable, viz. divergent, regime.

2. Transition in the Reduced Analogue System

As discussed in Section III, the $e^n$ rule is an empirically based rule whereby if the disturbance whose frequency yields the maximum amplification factor at transition is assumed to cause transition, then it is found that the value of the amplification factor itself is reasonably constant and corresponds to $n \simeq 9$. This result is extraordinary in that it is difficult to justify the linear theory in connection with such large amplification rates. Now consider this criterion in the reduced analogy system for divergence in the case of truly parallel flow where the amplification factor is constant in the flow direction and given by

$$-\int_{x_{N.S.}}^{x_T} \alpha_i dx = -\alpha_i \Delta x = n$$

where the subscripts N.S. and T refer to neutral stability and transition respectively. From Table 2 an instability corresponding to a given amplification factor is analogous to the response of the mechanical system at the frequency given by $\Omega = i\alpha = i(i\alpha_i) = -\alpha_i = n/\Delta x$. In other words, in the analogue system the $e^n$ rule is not to be interpreted in terms of a critical amplification factor, but rather in terms of a critical frequency.

3. Coupling of Disturbances to the Boundary Layer in the Reduced Analogue System

Generally transition is assumed to require both wavenumber and frequency (or phase velocity) matching in the flow direction with a potential disturbance. However, since the analogue in Table 2 has been reduced to the zero x-direction wavenumber component of the response of the plating structure by setting $k_x=0$, wavenumber matching between the structure and a potential disturbance merely requires that the disturbance exhibit a zero x-direction wavenumber component in its spectrum. In fact, since the dimensionality of the reduced analogue is that of a beam (in the y-z plane) rather than a plate, the condition of stability is a purely local phenomenon with no coupling in the x, or flow, direction. A resulting instability might well be called a "turbulent spot."

The eigenvalues of the Orr-Sommerfeld equation that are modelled by this reduced analogue may correspond to the category of disturbance generally referred to as Class C, since for this class there is apparently no dependence of the amplification rates on wavenumber.[9]

It is also of interest to consider the behavior of the reduced analogue structure for large values of y. Here the foundation impedance becomes uniform since U(y) approaches the free stream velocity $U_\infty$. The resulting structure, that of a uniform beam resting on a uniform foundation, will exhibit a cut-off along the y axis, i.e., normal to the boundary layer, if the foundation is stiffness-like. This frequency may be calculated from Eq. 11 by setting $k_y$, the y-direction wavenumber component of the solution, equal to zero as well as $k_x = 0$. This yields the cut-off frequency for large y

$$\Omega_{\text{cut-off}} = -\beta^3 Z_M^*$$

with the requirement that $Z_M^*$ be negative. From Table 2 it is seen that the analogue for divergence requires that $\underset{y\to\infty}{L}\{Z_M^*(y)\} = -(1 - c)$ and thus for potential disturbances with phase velocities typically associated with transition, viz. $c < 1$, $Z_M^*$ is indeed negative. Therefore for a divergent instability with $c < 1$ a cut-off frequency exists and it increases with increasing slenderness ratio, i.e., Reynolds number, such that for practical parameters this form of instability can only be excited by a disturbance in the near-field of the boundary layer.

## 4. Submarine Structure-Borne Noise in the Analogue System

Throughout this report, and indeed throughout most of the relevant published literature, destabilization is assumed to require a coincidence between the disturbance and those eigenvalues of the Orr-Sommerfeld equation that imply wave amplification. In other words, for example, the possibility of a phase velocity or wavenumber conversion mechanism within, or in the immediate vicinity of, the boundary layer is generally excluded.

However, the analogue system, and in particular the plate, allows two different characteristic modes of wave propagation. That is, for a given frequency, the characteristic equation, being fourth order as is the Orr-Sommerfeld equation, yields two branches corresponding to two separate wavelengths and phase velocities. (The effect of the foundation will be to modify these modes and most likely add an additional characteristic mode, or modes.) Nevertheless, in general for a given excitation, or disturbance, these two modes are coupled. In other words, within the analogue system itself is an inherent mechanism for coupling between relatively long and short wavelength motions. For example, in some circumstances, the mechanism for inducing relatively short wavelength Tollmien-Schlichting instabilities into a boundary layer by means of relatively long wavelength acoustic or structure-borne disturbances may be a manifestation of this (linear) phenomenon. To be precise the coupling would require coincidence between the disturbance and one of the stable eigenvalues of the Orr-Sommerfeld equation, i.e., one that implies wave attenuation, which would then be inherently coupled within the boundary layer itself to an unstable mode. The extent to which this represents a potential destabilizing mechanism in connection with structure-borne noise in submarine hulls is beyond the scope of this study.

## APPENDIX A: THE RESULTS OF A LINEAR STABILITY ANALYSIS INTO THE EFFECTS OF A BOUNDARY WHICH DISPLAYS FLEXURAL-LIKE VIBRATIONS

### 1. Introduction

Presented in this appendix are the results of a study[14] in which the inhomogeneous Orr-Sommerfeld equation is solved for the case of a traveling wave propagating along the boundary with constant amplitude, and real wave number and frequency. In other words

$$v(x,o) = -i\omega\zeta_o \exp[ik(x - ct)] = -i\omega\zeta_o \exp[i(kx - \omega t)] \tag{A1}$$

where $v(x,o)$ is the velocity imposed, and normal to, the boundary $y=0$ and $\zeta_o$ is displacement amplitude. The results are interpreted in terms of parameters relevant to the problem of structure-borne noise in submarine hull plating.

### 2. Results of Analysis

Using an approach analogous to that used by Benjamin[13] for the homogeneous case Becker is able to obtain explicit expressions for the wall pressure and for the exchange of energy between the boundary and the adjoining fluid for certain asymptotic situations characterized by large or small values of the parameter

$$b = \frac{c}{\nu k} = \frac{c^2}{\nu\omega} \tag{A2}$$

For b small, these results resemble those of the planar Couette flow, which automatically excludes Tollmien-Schlichting instabilities, and is therefore uninteresting as well as unrealistic. In contrast, for large b, Tollmien-Schlichting waves can arise.

The inviscid solution $\phi_1$ of the Orr-Sommerfeld equation yields the time-averaged power flow per unit boundary area, from the wall into the fluid:

$$E = -\frac{\rho c}{2k}\dot{\xi}_o^2 \mathrm{Im}[\phi_1'(o)] \quad , \quad b\to\infty \tag{A3}$$

where $\rho$ refers to the density of water. In Eq. A3, $\dot{\xi}_o$ is the velocity amplitude of the boundary, and $\mathrm{Im}[\phi_1'(o)]$ the imaginary component of $\phi_1'$ evaluated on the boundary. The energy flow associated with the viscid solution, being small for large b, need not be discussed. In view of Eq. A3, only complex or imaginary values of $\phi_1'(o)$ give rise to energy exchange. This restriction is satisfied when $U_\infty > c$, i.e. when a critical layer ($U(y) = c$) exists. This requirement is also obtained from Benjamin.[13] For $k\delta << 1$, i.e., for all situations of practical interest, it is found that

$$\phi_1'(o) = -\pi\omega^2 \left(1 - \frac{U_\infty}{c}\right)^4 \frac{U_c''}{(U_c')^3} \tag{A4}$$

where $U_c''$ and $U_c'$ are derivatives of $U(y)$ evaluated at the critical layer. Combining Eqs. A3 and A4, one obtains

$$E = \frac{\pi}{2}\rho c^2 \dot{\xi}_o^2 \omega \left(1 - \frac{U_\infty}{c}\right)^4 \frac{U_c''}{(U_c')^3} \quad ; \quad c<U_\infty \quad , \quad k\delta<<1 \quad , \quad b\to\infty \tag{A5}$$

For profiles characterized by $U_c'' < 0$, $\phi_1'(o) > 0$, a negative value of E results. This implies energy flow from the fluid into the boundary which is characteristic of all inviscid stable solutions. For $c > U_\infty$, $\phi_1'(o)$ is real, and $E = 0$.

Now, however, consider the case where the parameters $\omega$ and c in Eq. A1 match the eigenvalues of the homogeneous equation which implies $c < U_\infty$. Under these resonant or coincident conditions, for the Blasius profile

$$\phi_1'(o) = k(b_c/2)^{\frac{1}{2}} \quad , \quad b_c \to \infty \tag{A6}$$

where $b_c$ is a Reynolds number, Eq. A2, corresponding to neutral stability of Tollmien-Schlichting waves, say, $\omega_c$ and $k_c$. Since Eq. A6 is positive, this yields a negative value of E from Eq. A3, i.e. energy flow from the liquid to the wall. This appears to be consistent with Benjamin's solution, whereby Class A instabilities, i.e. Tollmien-Schlichting waves, are enhanced if the boundary is damped.

Asymptotic solutions for both $k\delta << 1$ and $k\delta >> 1$ indicate that energy flows from the fluid into the boundary for large Reynolds numbers. Becker speculates that this is a property of all velocity profiles which are stable in the absence of viscosity, e.g. the Blasius profile. For unstable profiles, i.e. profiles displaying an "inflexion" point (or a point where dU/dy becomes negative) a different situation arises. Here energy can flow in either direction. For $k\delta << 1$ and $k\delta >> 1$, $\phi_1'(o) < 0$ for $U_c'' > 0$, which makes Eq. A3 positive. (The significance of this situation to hull vibrations is discussed in Section IV. B.)

## APPENDIX B: AN ASYMPTOTIC LINEAR STABILITY ANALYSIS APPLICABLE TO DISTURBANCES AND BOUNDARY CONDITIONS ASSOCIATED WITH SUBMARINE HULL PLATING

### 1. Introduction

This appendix details the asymptotic solution to the Orr-Sommerfeld equation subject to the assumptions listed below,

a. $R \gg 1$, i.e., Reynolds numbers are large.

b. $c/U_\infty \gg 1$, i.e., the phase velocity of the potential disturbance to the laminar boundary layer is much greater than the free stream flow velocity.

Both assumptions are uniformly valid for practical problems with submarine speeds and vibration spectra associated with propagating flexural waves. The results of this analysis are applicable to the physical situations depicted in Fig. 5 as Items 3 and 4, viz., situations when the disturbance is introduced into the boundary layer other than at the boundary.

### 2. Analysis

For completeness, the Orr-Sommerfeld equation is redefined below

$$\phi^{IV}(y)-2\alpha^2\phi''(y)+\alpha^4\phi(y) = i\alpha R\{(U-c)[\phi''(y)-\alpha^2\phi(y)]-U''\phi(y)\} \tag{B1}$$

$\phi(y)$ is defined by the perturbation stream function $\Phi(x,y)$ which is assumed to be of the form

$$\Phi(x,y) = \phi(y)\ \exp[i\alpha(x - ct)] \tag{B2}$$

x and y are cartesian coordinates parallel and normal to the direction of flow. R is the Reynolds number. All equations are in non-dimensional form whereby lengths are normalized to the boundary layer thickness $\delta$; velocities to the free stream velocity $U_\infty$, time by $\delta/U_\infty$ and pressure by $\rho U_\infty^2$ where $\rho$ is the fluid density.

The stability problem is thus represented by the solution to Eq. B1 subject to appropriate boundary conditions.

Consider the case of parallel flow over a flat boundary of infinite extent. The appropriate conditions at the outer edge of the boundary layer can be expressed by

$$\phi(\infty) = \phi'(\infty) = 0 \tag{B3}$$

At the boundary, continuity of velocity tangential to the boundary dictates that[9]

$$\begin{aligned} &(1 - c/Z_t)[\phi'(0) + (U'(0)/c)\phi(0)] \\ &\quad + i(Z_t \alpha R)^{-1}[\phi'''(0) - \alpha^2\phi'(0)] = 0 \end{aligned} \tag{B4}$$

where $Z_t$ is the tangential impedance of the boundary.

For the fourth boundary condition continuity of normal velocity is prescribed in terms of the normal impedance of the boundary

$$p(0)/[-i\alpha\phi(0)] = -Z_n \tag{B5}$$

where p(0) is the perturbation pressure at the boundary.

Boundary conditions (B4) and (B5) pertain to the modifying effect of a finite impedance hull on disturbances in the laminar boundary layer. The x-momentum equation can be conveniently used to express p(0) in terms of the stream function*.

$$p(0) = -i[\phi'''(0) - \alpha^2\phi'(0)]/(\alpha R) + c\phi'(0) \pm U'(0)\phi(0) \tag{B6}$$

Consider the behavior of Eq. B1 for large values of R, i.e. R >> 1. In this range, Eq. B1 can be satisfied via two mechanisms:

*This expression differs from that used by Benjamin[13] which was derived using the y-momentum equation and is of integral form.

a. $\phi$ exhibits large gradients thus making the left hand side of Eq. B1 comparable to the right hand side or

b. $|(U - c)(\phi'' - \alpha^2\phi) - U''\phi| << 1$.

Mechanism (a) is a cause of considerable difficulty in developing numerical integration solutions to Eq. B1.

Eq. B1 with $R >> 1$ is of the form of a singular perturbation problem in that the highest ordered derivative term is multiplied by a small number, namely $\varepsilon \equiv R^{-1}$. This can result in large gradients being exhibited by the solution in order to satisfy imposed boundary constraints. The classical approach to such problems is to define an "inner" solution that is valid within the vicinity of those large gradients, and to "match" this solution with an "outer" solution that is valid beyond this region.[19] The particular problem under consideration is somewhat degenerate in that the first order outer solution which is obtained by setting $R = \infty$ (or $\varepsilon = 0$) in Eq. B1, is identically zero assuming the quantity $c\alpha^2 << 1$.[20] This means that the inner solution must be uniformly valid across the boundary layer.

In order to obtain the appropriate inner solution, Eq. B1 is subjected to a change in the independent variable in the form $\eta = y/\varepsilon^n$. The resulting equation is then solved via a power series in $\varepsilon$. We will only deal here with the first order approximation.

If the term $(U - c)$ changes sign within the domain of the boundary layer the appropriate transformation is given by $n = 1/3$.[20] This yields a solution in terms of Bessel functions of order 1/3, and in turn the familiar Tietjens function. However, for the case at hand, where $|U/c| << 1$ throughout the boundary layer, a more convenient transformation results when $n = 1/2$. Thus the "stretch" variable $\eta$ is defined by

$$\eta = y/\varepsilon^{\frac{1}{2}} \tag{B7}$$

Substituting Eq. B7 into Eq. B1 and allowing $\varepsilon \Rightarrow 0$ yields the equation

$$\phi_{\eta\eta\eta\eta}(\eta) + ic\phi_{\eta\eta}(\eta) = 0\ (\varepsilon^2) \doteq 0 \tag{B8}$$

The general solution to Eq. B8 is given by

$$\phi = A\sin[(1 + i)a\eta] + B\cos[(1 + i)a\eta] + C + D\eta \tag{B9}$$

with $a = (c/2)^{\frac{1}{2}}$.

However, Eq. B3 represents a severe constraint on Eq. B9 and yields a non-trivial solution only if

$$C = D = 0 \tag{B10}$$

and either

$$B = -iA \tag{B11a}$$

or

$$\mathrm{Re}[a] = -\mathrm{Im}[a] \tag{B11b}$$

Condition B11b implies that c is imaginary and negative and this, in turn, implies a stable solution.

Thus, the problem is reduced to the task of determining if a solution in the form

$$\phi = A[\sin(1 + i)a\eta - i\cos(1 + i)a\eta] \tag{B12}$$

is compatible with the boundary conditions represented by Eqs. B4 and B5 and, if compatible, whether an unstable mode is possible. The fact that Eq. B12 exhibits only one free constant, A, requires a compatibility condition on the boundary conditions themselves.

A solution exists only if

$$Z_t = -Z_n\phi(0)/[(U'(0)/c)\phi(0) + \phi\eta(0)/\varepsilon] \tag{B13}$$
$$\approx 0 \quad \varepsilon \to 0$$

Now, subjecting Eq. B6 to the coordinate transformation given by Eq. B7 yields

$$\varepsilon p(0) = -i[\phi_{\eta\eta\eta}(0) - \alpha^2\varepsilon^2\phi_\eta(0)] + c\phi_\eta(0) + \varepsilon U'(0)\phi(0) \tag{B14}$$

Using Eq. B8 and dropping $\varepsilon$ terms yields the result

$$p(0) = U'(0)\phi(0) \tag{B15}$$

which when substituted into Eq. B5 gives

$$Z_n = -iU'(0)/\alpha \tag{B16}$$

If Eq. B16 is to be interpreted in dimensional form, then the right hand side of the equation must be multiplied by $\rho$.

3. Discussion of Results

Eqs. B13 and B16 give the values of the boundary impedances that are compatible with a neutrally stable condition. It should be noted that for a disturbance traveling in the same direction as the flow, i.e. (Re(a) > 0), the required normal impedance is inertial. However, for a disturbance traveling opposite to the flow, the impedance is stiffness-like. The effect of energy dissipation within the boundary can also be analyzed from Eq. B16. Boundary dissipation would correspond to a positive real component of $Z_n$. However, from Eq. B16 this would require a negative imaginary component of $\alpha$ which implies an unstable condition when $Re(\alpha) > 0$ and a stable condition when $Re(\alpha) < 0$ (Eq. B2). Therefore the effect of boundary dissipation is destabilizing for potential disturbances traveling in the direction of flow and stabilizing when the disturbance travels against the flow. More importantly condition B13 which is required for an instability is not satisfied by present-day submarine hull construction.

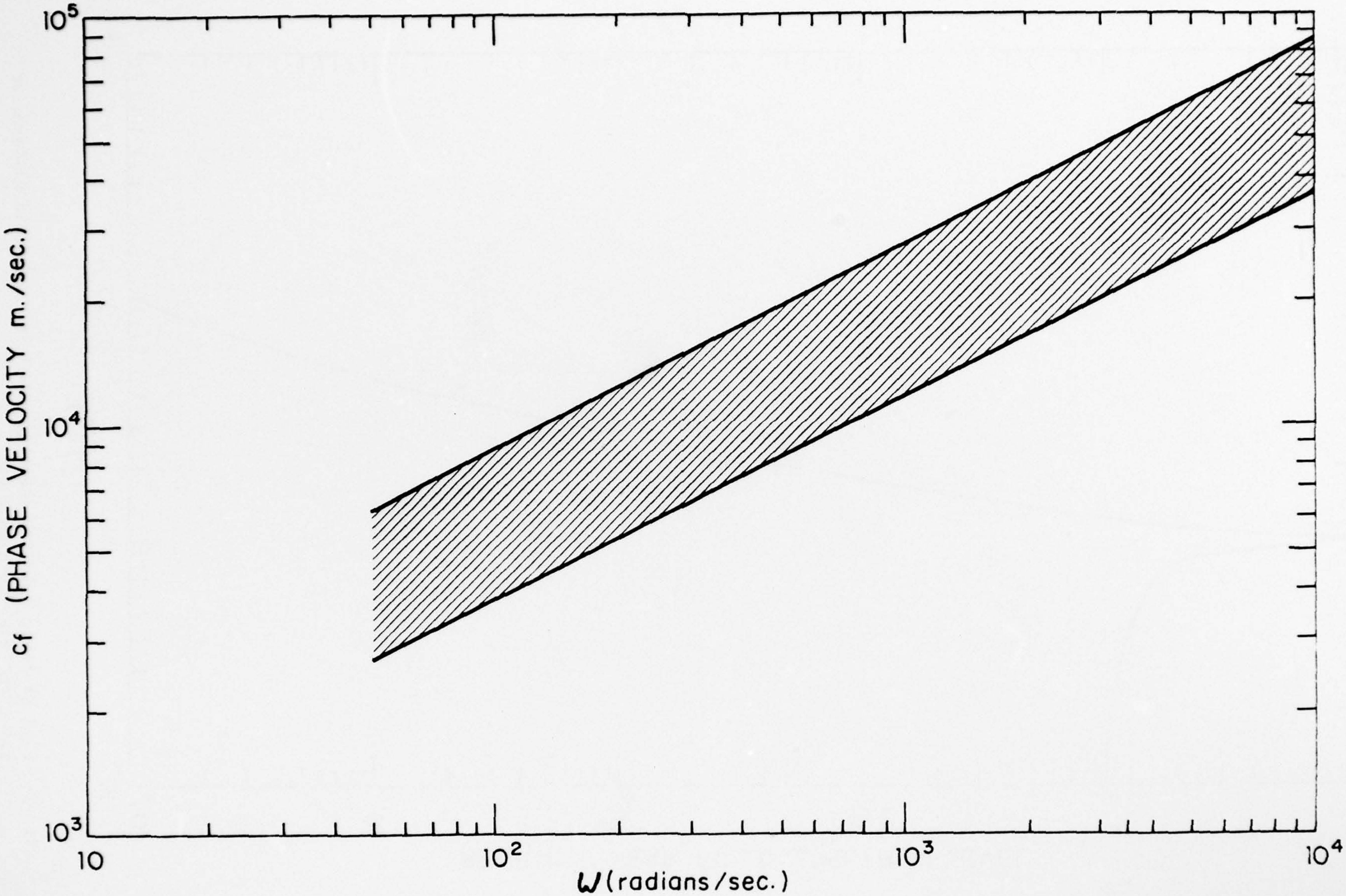


Fig. 1 Enveloped phase velocities vs. frequency plot for freely propagating flexural waves in submarine hull plating.

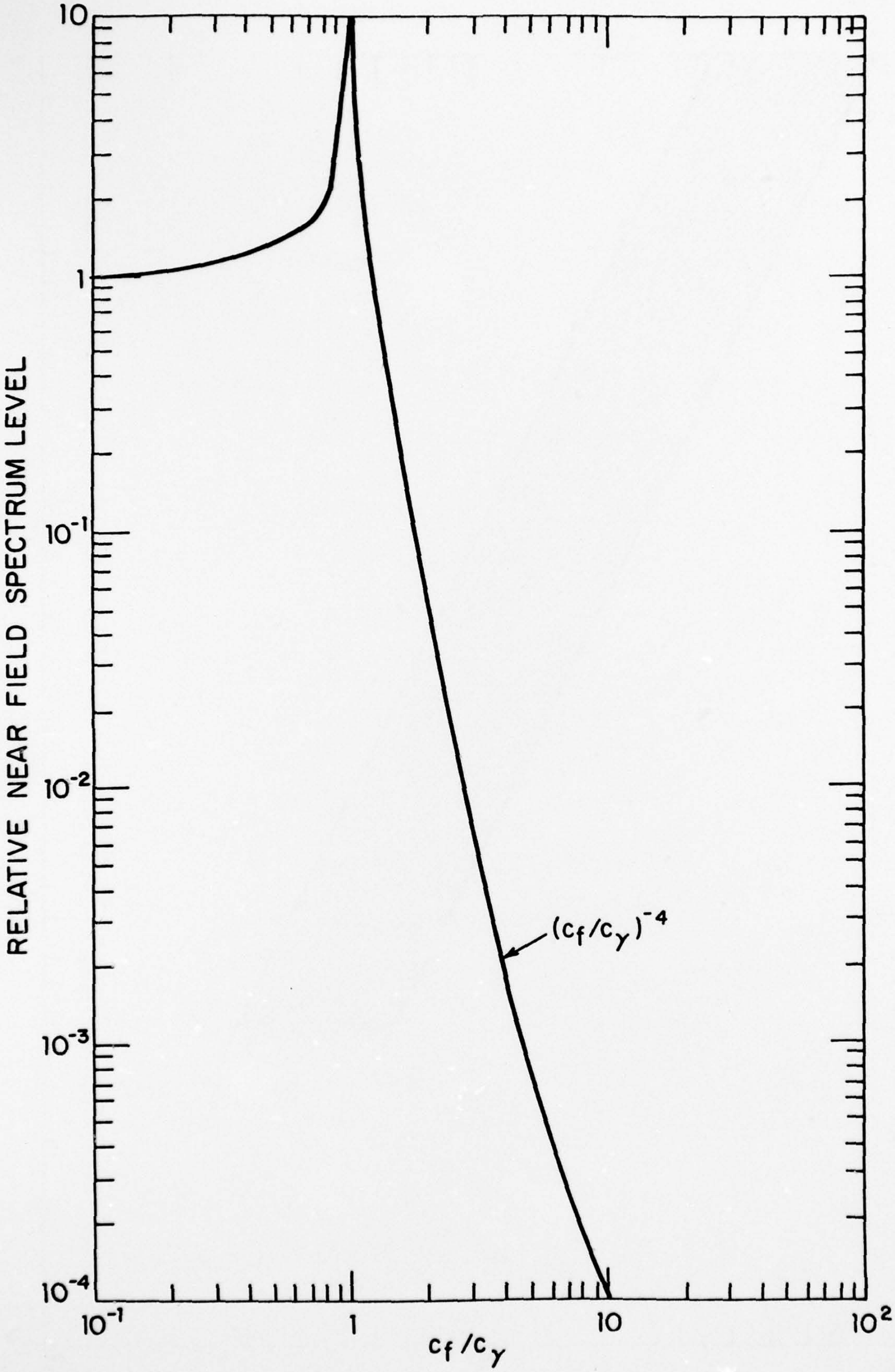


Fig. 2 Plot of phase velocity spectrum level in near-field disturbance for structural damping $\eta$ = 0.1.

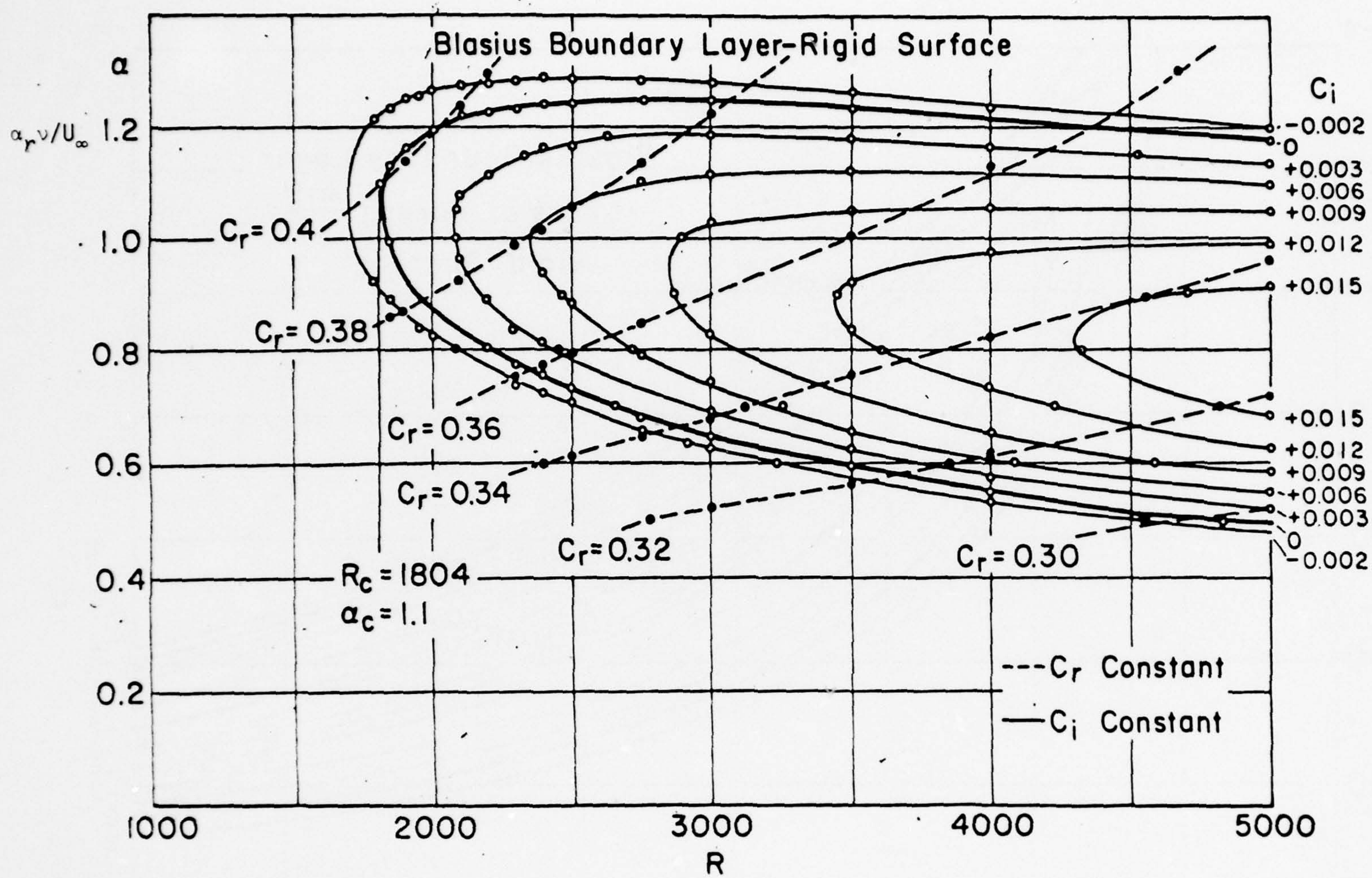


Figure 3 **Eigenvalues $c_r$, $c_i$ for the Blasius Boundary Layer *over* a Rigid Surface** ($c_r$ and $c_i$ normalized to $U_\infty$). Reproduced from Reference 9.

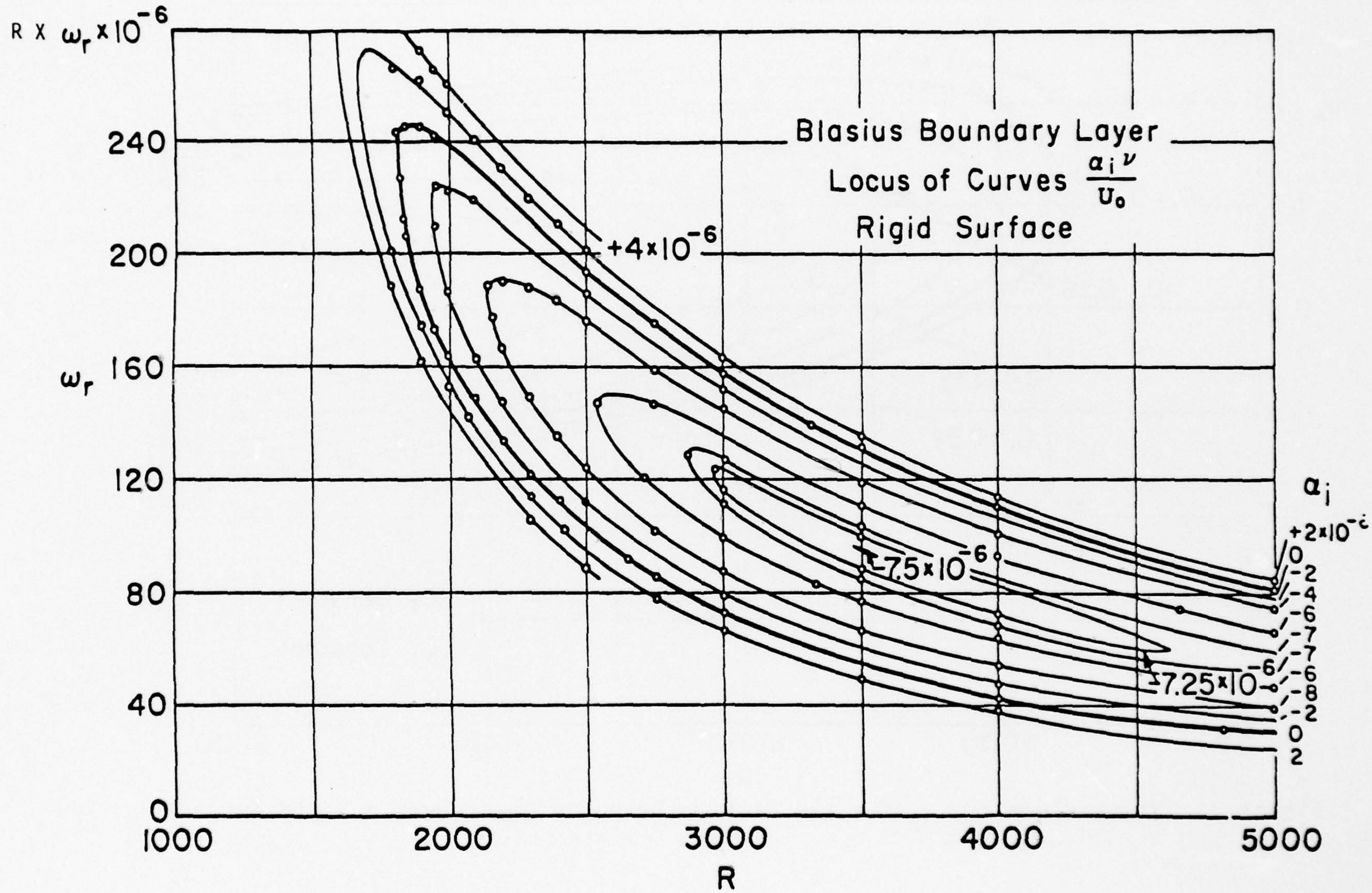


Figure 4 Spatial Amplification Rates for the Blasius Boundary Layer over a Rigid Surface $[\omega_r \times R = (c_r/U_\infty)(\alpha_r \nu/U_\infty)]$. Reproduced from Reference 9.

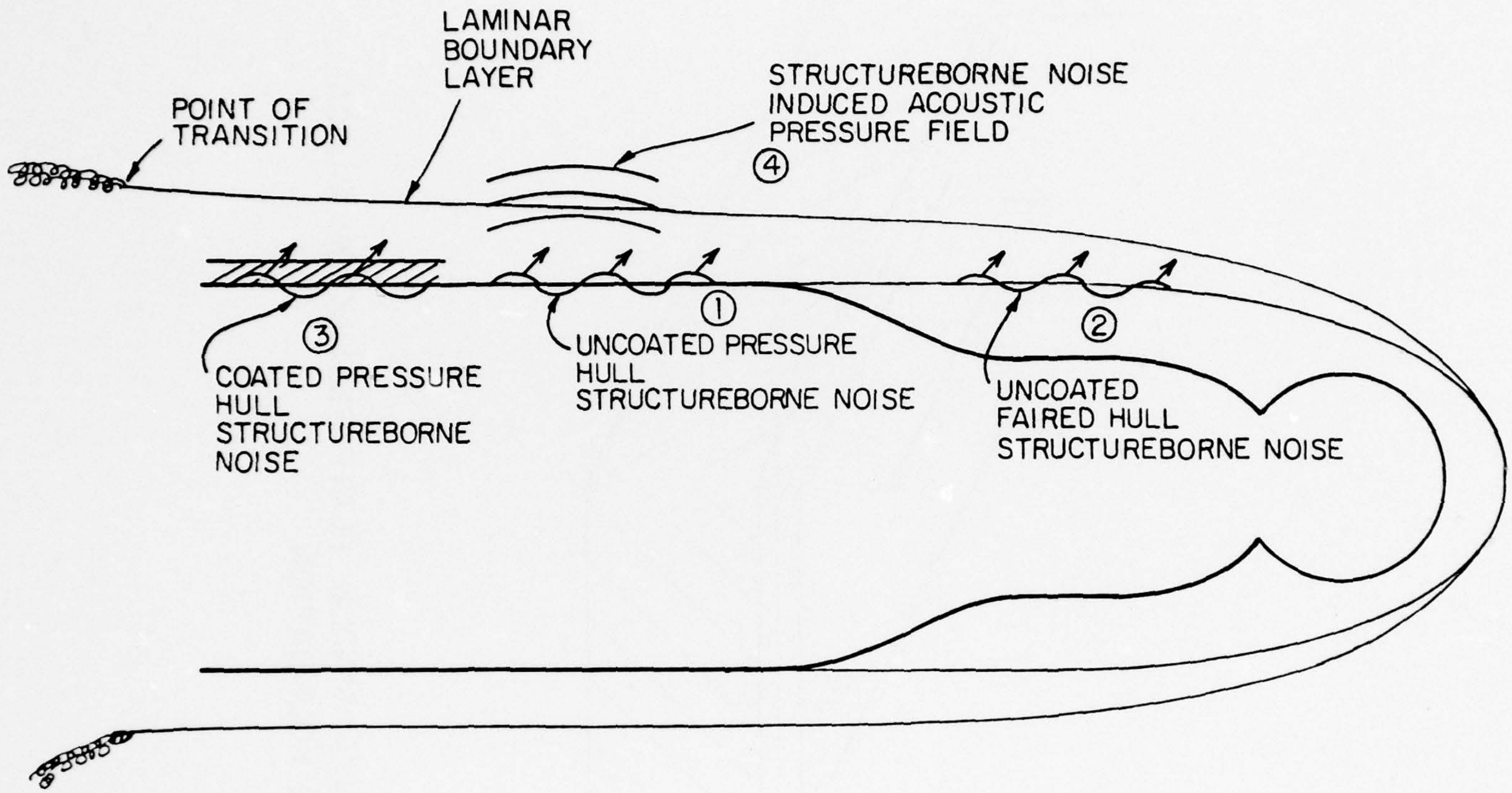


Fig. 5 Potential Destabilizing Effects of Structure-Borne Noise on Hypothetical Laminar Flow Submersible

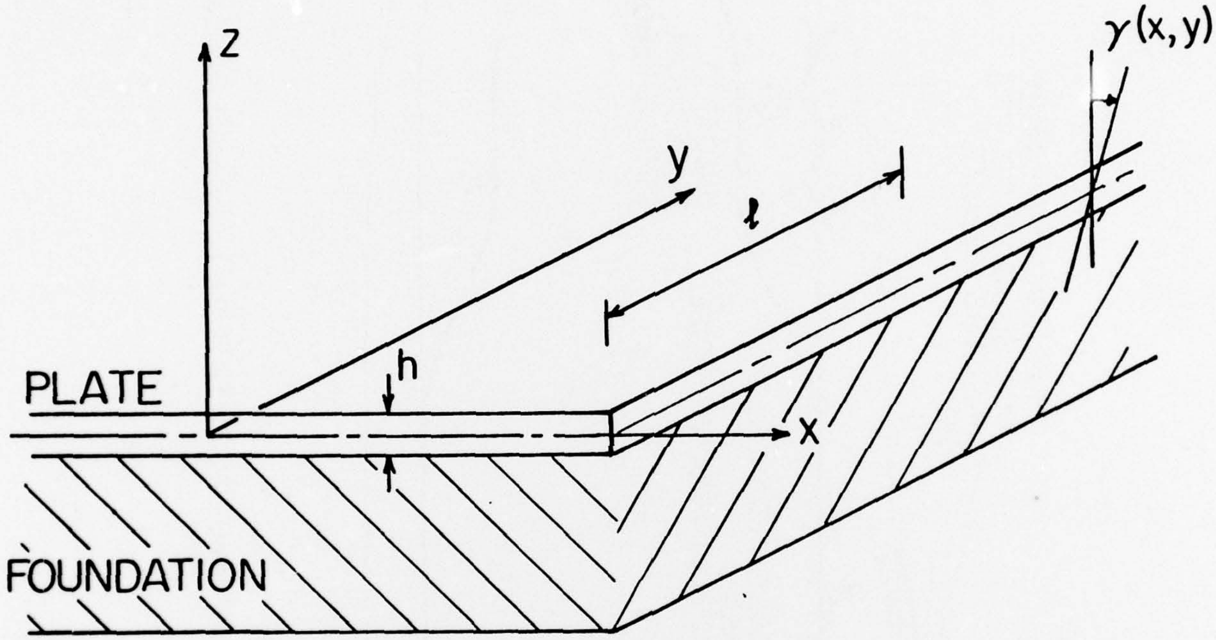


FIG. 6 TIMOSHENKO-MINDLIN PLATE RESTING ON A LOCALLY REACTING FOUNDATION

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